NON-PARAMETRIC PROCEDURES FOR COMPARING A SET OF RESPONSE CURVES

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ABSTRACT

Parametric models may or may not be available to a researcher who wishes to compare a number of response curves. Two non-parametric procedures are described which will allow a researcher to compare a set of response curves of any shape or form. The procedures are simple and straightforward. No curve fitting is involved and the computational procedures are simple.

Key words: Chi square; Mean rank sum; parametric model.

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INTRODUCTION

A frequent consulting question involves the comparison of a set of response curves. Sometimes the consultee has little statistical or computer expertise. Also, the formulation of a mathematical equation for the response curve may be beyond the expertise of the consultee or the consulting statistician. Of course, one could always use polynomial regression but this is a default option and may not be meaningful, e. g., if the response reaches an asymptote. Thus, a very simple procedure would be a welcome option for them. Two rather simple procedures are presented here for comparing response curves.

Response curves take many shapes and forms. Some response curves may take the same path for a period of time and then diverge such as for example in tolerance studies. Others may reach a peak and then decline, sometimes sharply. The point of decline may be different for each response curve.

TWO RESPONSE CURVES

Suppose the following data set were obtained for two sets of responses Y, dependent variable, for condition X, independent variable:

Set 1:	Y--2	4	8	10	11	12	13	13	13	14
	X--1	2	3	4	5	6	7	8	9	10
Set 2:	Y--6	8	10	11	10	13	13	14	15	15
	X--1	2	3	4	5	6	7	8	9	10

A graph of the responses is presented as Figure 1.

For the first method of comparing the two sets of responses, a table of the following nature is constructed for determining if a response in Set 1 is more than its counterpart in Set 2:

	Set 1	Set 2
More	11/2	8 1/2

The responses are tied for X = 7. Therefore, 1/2 is allocated to each set. If the two response curves were the same, the median value would be 5.5 since there are 10 pairs of responses. Thus, a simple chi square test may be used to compare the two response curves. The value of the one degree of freedom chi square is computed as follows:

$$\chi^2 = (1\ 1/2 - 5.5)^2/5.5 + (8\ 1/2 - 5.5)^2/5.5 = 4.55.$$

A second procedure is to rank the responses within each of the ten pairs and obtain a mean rank sum as follows:

Pair X	1	2	3	4	5	6	7	8	9	10	sum
Set 1	1	1	1	1	2	1	1.5	1	1	1	11.5
Set 2	2	2	2	2	1	2	1.5	2	2	2	18.5

A mean rank test is computed as follows. The number of t-way ties needs to be computed. Then R1 = 11.5, R2 = 18.5, rj = 10 pairs, and v = 2 sets. There are nine (r1) one-way ties. There is one (r2) two-way tie, 1.5. There are no other ties. T1 = r1(t)(t - 1)(t + 1) = 9(1)(1 - 1)(1 + 1) = 0. T2 = 1(2)(2 - 1)(2 + 1) = 6. The correction for ties is computed as

$$1 - \sum_{i=1 \text{ to } 5} Ti/r(r^2 - 1) = 1 - 6/10(100 - 1) = 0.994.$$

The test statistic with one degree of freedom is computed as

$$\chi^2 = \{[12/rv(v + 1)]\sum_{j=1 \text{ to } 2} Rj^2 - 3r(v + 1)\} / 0.994$$

$$= \{[12/ 60][11.5^2 + 19.5^2] - 3(10)(2 + 1)\} / 0.994$$

$= \{94.9 - 90\} / 0.993 = 4.93.$

In practice, the correction for a few ties may be ignored as the correction will usually be close to one. The two tests agree fairly well.

SET OF RESPONSE CURVES

Consider the following data set for v = 4 response curves over 9 time periods, X = 1, 2, …, 9:

Time	1	2	3	4	5	6	7	8	9
Curve 1	2	4	8	10	12	14	14	14	14
Curve 2	6	8	10	11	11	12	12	13	14
Curve 3	1	2	3	5	7	9	9	9	9
Curve 4	0	1	2	3	2	4	2	3	4

A graphical presentation of these response curves is given in Figure 2. The following table of ranks within each value of X is computed next:

Time	1	2	3	4	5	6	7	8	9	Rj
Curve 1	3	3	3	3	4	4	4	4	3.5	31.5
Curve 2	4	4	4	4	3	3	3	3	3.5	31.5
Curve 3	2	2	2	2	2	2	2	2	2	18
Curve 4	1	1	1	1	1	1	1	1	1	9
Sum	10	10	10	10	10	10	10	10	10	90

The Rj are the sum of the ranks for each curve, j = 1, 2, 3, 4. The chi square statistic is computed as below.

$T1 = 34(1)(1 - 1)(1 + 1) = 0; T2 = 1(2)(2 - 1)(2 + 1) = 6.$

The correction for ties is the same one as computed above, i. e., 0.994. The chi square statistic with 4 - 1 = 3 degrees of freedom is

$$\chi^2 = [12/rv(v + 1)][\sum_{j = 1 \text{ to } 4} Rj^2] - 3r(v + 1)$$

$$= [0.0667][31.5^2 + 31.5^2 + 18^2 + 9^2] - 3(9)(4 + 1)$$

$$= 159.4 - 135 = 24.5.$$

The chi square value would be increased if the correction for ties were used. For responses of the nature of curves 3 and 4, this procedure will not detect a difference when crossovers of responses occur.

COMMENT

In many situations, the researcher only wishes to ascertain if the response curves differ and may not wish to know the mathematical formula for a response curve. In such situations, the above simple procedures allow for this to be accomplished in an easy and efficient manner. The procedure is easy to explain and understand when used in published articles.

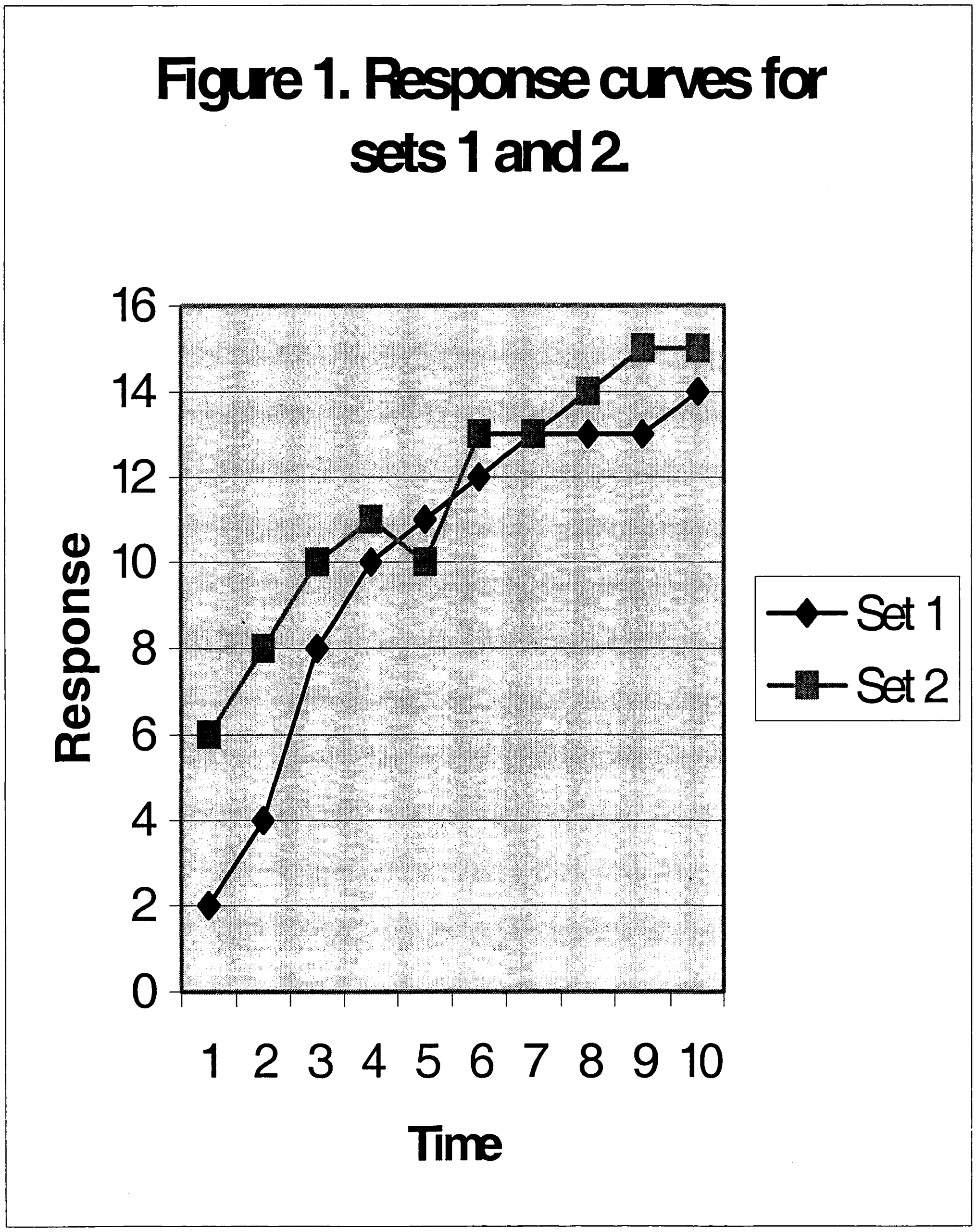

Figure 2. Response curves 1 to 4.